SAMPLE PAPER

issued by CBSE for Board Exams (2024-25)

Mathematics (041) - Class 12

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (with a total of **4 marks** for each Case-study / Source-based / Passage-based question).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

SECTION A

*(Question numbers 01 to 20 carry **1 mark** each.)*

*Followings are **multiple choice questions**. Select the correct option in each one of them.*

01. If for a square matrix A, $A.(\text{adj}.A)=\begin{bmatrix}2025 & 0 & 0\\ 0 & 2025 & 0\\ 0 & 0 & 2025\end{bmatrix}$, then the value of $|A|+|\text{adj}.A|$ is equal to

(a) 1 (b) $2025+1$ (c) $(2025)^2+45$ (d) $2025+(2025)^2$

02. Assume X, Y, Z, W and P are the matrices of order $2\times n, 3\times k, 2\times p, n\times 3$ and $p\times k$ respectively. Then the restriction on n, k and p so that $PY+WY$ will be defined are

(a) $k=3,\ p=n$ (b) k is arbitrary, $p=2$

(c) p is arbitrary, $k=3$ (d) $k=2,\ p=3$

03. The interval in which the function f defined by $f(x)=e^x$ is strictly increasing, is

(a) $[1,\infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

04. If A and B are non-singular matrices of same order with $\det(A)=5$, then $\det(B^{-1}AB)^2$ is equal to

(a) 5 (b) 5^2 (c) 5^4 (d) 5^5

05. The value of 'n', such that the differential equation

$x^n\dfrac{dy}{dx}=y(\log y-\log x+1)$; where $x,\ y\in R^+$ is homogeneous, is

(a) 0 (b) 1 (c) 2 (d) 3

06. If the points $(x_1, y_1), (x_2, y_2)$ and (x_1+x_2, y_1+y_2) are collinear, then x_1y_2 is equal to

(a) x_2y_1 (b) x_1y_1 (c) x_2y_2 (d) x_1x_2

07. If $A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $a+b+c=$

(a) 1 (b) 2 (c) 3 (d) 4

08. For any two events A and B, if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P(\bar{A} \mid \bar{B})$ equals

(a) $\frac{3}{8}$ (b) $\frac{8}{9}$ (c) $\frac{5}{8}$ (d) $\frac{1}{4}$

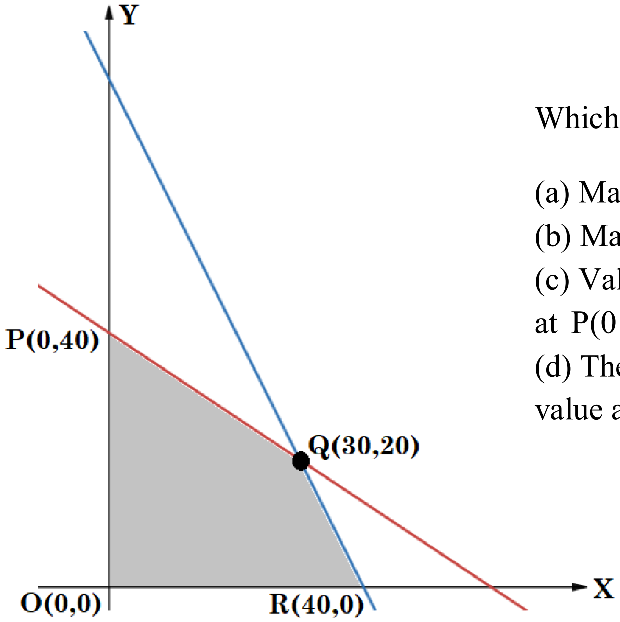
09. The value of α if the angle between $\vec{p} = 2\alpha^2\hat{i} - 3\alpha\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + \hat{j} + \alpha\hat{k}$ is obtuse, is

(a) $\mathbb{R} - [0, 1]$ (b) $(0, 1)$ (c) $[0, \infty)$ (d) $[1, \infty)$

10. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}| =$

(a) 3 (b) 4 (c) 5 (d) 8

11. For the linear programming problem (L.P.P.), the objective function is $Z = 4x + 3y$ and the feasible region determined by a set of constraints is shown in the graph:



Which of the following statements is true?

(a) Maximum value of Z is at $R(40, 0)$.
(b) Maximum value of Z is at $Q(30, 20)$.
(c) Value of Z at $R(40, 0)$ is less than the value at $P(0, 40)$.
(d) The value of Z at $Q(30, 20)$ is less than the value at $R(40, 0)$.

12. $\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}}$ equals

(a) $-\frac{1}{2x^2}\sqrt{1+x^4} + c$ (b) $\frac{1}{2x}\sqrt{1+x^4} + c$

(c) $-\frac{1}{4x}\sqrt{1+x^4} + c$ (d) $\frac{1}{4x^2}\sqrt{1+x^4} + c$

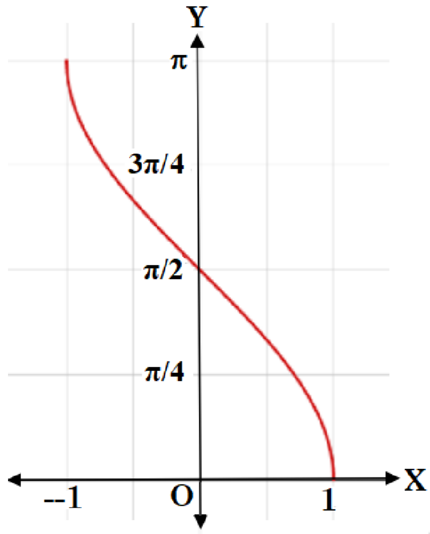
13. $\int_0^{2\pi} \operatorname{cosec}^7 x \, dx =$

(a) 0 (b) 1 (c) 4 (d) 2π

14. What is the general solution of the differential equation $e^{\frac{dy}{dx}} = x$?

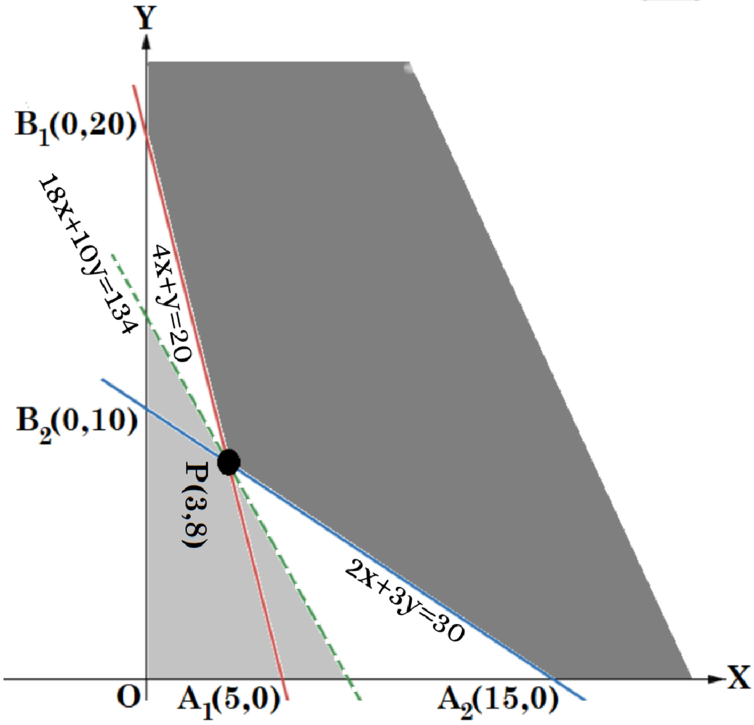
(a) $y = x\log x + c$ (b) $y = x\log x - x + c$ (c) $y = x\log x + x + c$ (d) $y = x + c$

15. The graph drawn below depicts



(a) $y = \sin^{-1} x$ (b) $y = \cos^{-1} x$ (c) $y = \text{cosec}^{-1} x$ (d) $y = \cot^{-1} x$

16. A linear programming problem (L.P.P.) along with the graph of its constraints is shown below.



*(**Note :** The figure is not to scale.)*

The corresponding objective function is: $Z = 18x + 10y$, which has to be minimized. The smallest value of the objective function Z is 134 and is obtained at the corner point $(3, 8)$.

The optimal solution of the above linear programming problem ____________.

(a) does not exist as the feasible region is unbounded

(b) does not exist as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region

(c) exists as the inequality $18x + 10y > 134$ has infinitely many points in common with the feasible region

(d) exists as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region

17. The function $f : \mathbb{R} \to Z$ defined by $f(x) = [x]$; where [.] denotes the greatest integer function, is

(a) continuous at $x = 2.5$ but not differentiable at $x = 2.5$
(b) not continuous at $x = 2.5$ but differentiable at $x = 2.5$
(c) not continuous at $x = 2.5$ and not differentiable at $x = 2.5$
(d) continuous as well as differentiable at $x = 2.5$

18. A student observes an open-air Honeybee nest on the branch of a tree, whose plane figure is parabolic shape given by $x^2 = 4y$. Then the area (in Sq. units) of the region bounded by parabola $x^2 = 4y$ and the line $y = 4$ is

(a) $\frac{32}{3}$ (b) $\frac{64}{3}$ (c) $\frac{128}{3}$ (d) $\frac{256}{3}$

Followings are ***Assertion-Reason based questions****.*

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A) :** Consider the function defined as $f(x) = |x| + |x-1|,\ x \in \mathbb{R}$. Then $f(x)$ is not differentiable at $x = 0$ and $x = 1$.

Reason (R) : Suppose f be defined and continuous on (a, b) and $c \in (a, b)$, then $f(x)$ is not differentiable at $x = c$ if $\lim_{h\to 0^-} \frac{f(c+h) - f(c)}{h} \neq \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$.

20. **Assertion (A) :** The function $f : \mathbb{R} - \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\} \to (-\infty, -1] \cup [1, \infty)$ defined by $f(x) = \sec x$ is not one-one function in its domain.

Reason (R) : The line $y = 2$ meets the graph of the function at more than one point.

SECTION B

(Question numbers 21 to 25 carry ***2 marks*** *each.)*

21. If $\cot^{-1}(3x+5) > \frac{\pi}{4}$, then find the range of the values of x.

22. The cost (in rupees) of producing x items in factory, each day is given by

$$C(x) = 0.00013x^3 + 0.002x^2 + 5x + 2200$$

Find the marginal cost when 150 items are produced.

23. Find the derivative of $\tan^{-1} x$ with respect to $\log x$; where $x \in (1, \infty)$.

OR

Differentiate the given function with respect to x : $(\cos x)^x$; where $x \in \left(0, \frac{\pi}{2}\right)$.

24. If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{b} + \lambda\vec{c}$ is perpendicular to $\vec{a}$, then find the value of λ.

OR

A person standing at $O(0, 0, 0)$ is watching an aeroplane which is at the point $A(4, 0, 3)$. At the same time he saw a bird at the point $B(0, 0, 1)$. Find the angles which $\overrightarrow{BA}$ makes with the x, y and z axes.

25. The two co-initial adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find its diagonals and use them to find the area of the parallelogram.

SECTION C

*(Question numbers 26 to 31 carry **3 marks** each.)*

26. A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.

27. According to a psychologist, the ability of a person to understand spatial concepts is given by $A = \frac{1}{3}\sqrt{t}$, where t is the age in years, $t \in [5, 18]$. Show that the rate of increase of the ability to understand spatial concepts decreases with age in between 5 and 18.

28. An ant is moving along the vector $\vec{l_1} = \hat{i} - 2\hat{j} + 3\hat{k}$. Few sugar crystals are kept along the vector $\vec{l_2} = 3\hat{i} - 2\hat{j} + \hat{k}$ which is inclined at an angle θ with the vector $\vec{l_1}$. Then find the angle θ. Also find the scalar projection of $\vec{l_1}$ on $\vec{l_2}$.

OR

Find the vector and the Cartesian equation of the line that passes through $(-1, 2, 7)$ and is perpendicular to the lines $\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 7\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$.

29. Evaluate : $\int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$; where $x > 1$.

OR

Evaluate : $\int_0^1 x(1-x)^n dx$; where $n \in N$.

30. Consider the following Linear Programming Problem.

Minimize $Z = x + 2y$

Subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.

Show graphically that the minimum of Z occurs at more than two points.

31. The probability that its rains today is 0.4. If its rains today, the probability that it will rain tomorrow is 0.8. If it does not rain today, the probability that it will rain tomorrow is 0.7. If

P_1 : denotes the probability that it does not rain today;

P_2 : denotes the probability that it will not rain tomorrow, if it rains today;

P_3 : denotes the probability that it will rain tomorrow, if it does not rain today;

P_4 : denotes the probability that it will not rain tomorrow, if it does not rain today.

(i) then find the value of $P_1 \times P_4 - P_2 \times P_3$.

(ii) then calculate the probability of raining tomorrow.

OR

A random variable X can take all non-negative integral value and the probability that X takes the value r is proportional to 5^{-r}. Find $P(X < 3)$.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. Draw the rough sketch of the curve $y = 20\cos 2x$; where $\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$.

Using integration, find the area of the region bounded by the curve $y = 20\cos 2x$ from the ordinates $x = \frac{\pi}{6}$ to $x = \frac{\pi}{3}$ and the x-axis.

33. The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$; (where $0 \leq x \leq 14$ and $a, b, c \in \mathbb{R}$ and $a \neq 0$) with respect to a XY-coordinate system in the vertical plane. The ball passes through the points $(2, 15)$, $(4, 25)$ and $(14, 15)$. Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also, find the equation of the path traversed by the ball.

34. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=|x|^3$, show that $f''(x)$ exists for all real x and find it.

OR

If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ is a constant independent of a and b.

35. Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$$\vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\lambda(7\hat{i}-6\hat{j}+\hat{k}) \text{ and } \vec{r}=(3\hat{i}+5\hat{j}+7\hat{k})+\mu(\hat{i}-2\hat{j}+\hat{k}),$$

where λ and μ are parameters.

OR

Find the image of the point $(1, 2, 1)$ with respect to the line $\dfrac{x-3}{1}=\dfrac{y+1}{2}=\dfrac{z-1}{3}$. Also find the equation of the line joining the given point and its image.

SECTION E

*(Question numbers 36 to 38 carry **4 marks** each.)*
*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** (i) and (ii) of **2 marks** each.*

36. CASE STUDY I :

Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension 25 cm by 40 cm to make container packets without top. Let x cm be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps.
Based on the above information, answer the following equations.

(i) Express the volume (V) of each container as function of x only.

(ii) Find $\dfrac{dV}{dx}$.

(iii) For what value of x, the volume of each container is maximum?

OR

(iii) Check whether V has a point of inflection at $x=\dfrac{65}{6}$ or not?

37. CASE STUDY II :

An organization conducted bike race under 2 different categories - boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let $B=\{b_1, b_2, b_3\}$, $G=\{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.
Ravi decides to explore these sets for various types of relations and functions.
On the basis of the above information, answer the following questions.

(i) Ravi wishes to form all the relations possible from B to G. How many such relations are possible?
(ii) Write the smallest equivalence relation on G.
(iii) Ravi defines a relation from B to B as $R_1 = \{(b_1, b_2), (b_2, b_1)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes (A) reflexive but not symmetric, (B) reflexive and symmetric but not transitive.

OR

(iii) If the track of final race (for the biker b_1) follows the curve $x^2 = 4y$; where $0 \le x \le 20\sqrt{2}$ and $0 \le y \le 200$, then state whether the track represents a one-one and onto function or not. Justify your answer.

38. CASE STUDY III :

Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage-II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously). Then two birds flew back from Cage-II to Cage-I (simultaneously).
Assume that all the birds have equal chances of flying.
On the basis of the above information, answer the following questions.

(i) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, then find the probability that the owl is still in Cage-I .
(ii) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II.

🕮 Detailed Solutions for CBSE Sample Paper (2024-25)

SECTION A

01. (d) For a square matrix A of order $n\times n$, we have $A.(adj.A)=|A|I_n$, where I_n is the identity matrix of order $n\times n$.

So, $A.(adj.A)=\begin{bmatrix}2025&0&0\\0&2025&0\\0&0&2025\end{bmatrix}=2025\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=2025\ I_3$

$\Rightarrow |A|\ I_3=2025\ I_3$ i.e., $|A|=2025$.

So, $|adj.A|=|A|^{3-1}=(2025)^2$.

Hence, $|A|+|adj.A|=2025+(2025)^2$.

02. (a) Order of $P_{p\times k}Y_{3\times k}$ will be $p\times k$; order of $W_{n\times 3}Y_{3\times k}$ will be $n\times k$.

Also when PY is defined, then we must have $k=3$ ($\because$ no. of columns in P = no. of rows in Y).

Hence, order of PY will be $p\times 3$; order of WY will be $n\times 3$.

For $PY+WY$ to exist, we must have order of $PY=$ order of WY.

That means, we must compare the respective orders of PY and WY to get $p=n$.

03. (c) $f(x)=e^x \quad \Rightarrow f'(x)=e^x$

Note that, $e^x>0\ \forall x\in\mathbb{R}$.

In the domain ($\mathbb{R}$) of the function, $f'(x)>0$.

Hence, the function is strictly increasing in $(-\infty,\infty)$.

04. (b) $\det(B^{-1}AB)^2=|B^{-1}AB|^2=\left[|B^{-1}||A||B|\right]^2=\left[|B|^{-1}|A||B|\right]^2=|A|^2=5^2$.

05. (b) $x^n\dfrac{dy}{dx}=y(\log y-\log x+1)$ can be rewritten as $x^n\dfrac{dy}{dx}=y\left(\log_e\dfrac{y}{x}+\log_e(e)\right)$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{y}{x^n}\left\{\log_e\left(e\times\dfrac{y}{x}\right)\right\}=f(x,\ y)$ *say*.

Hence, $f(x,\ y)$ will be a homogeneous function of degree 0, if $n=1$.

Note that, a differential equation of the form $\dfrac{dy}{dx}=f(x,\ y)$ is said to be homogeneous, if $f(x,\ y)$ is a homogeneous function of degree 0.

06. (a) When the points $(x_1,\ y_1)$, $(x_2,\ y_2)$ and $(x_1+x_2,\ y_1+y_2)$ are collinear, then we must have

$$\begin{vmatrix}x_1&y_1&1\\x_2&y_2&1\\x_1+x_2&y_1+y_2&1\end{vmatrix}=0$$

Expanding along C_3, we get

$1.(x_2y_1+x_2y_2-x_1y_2-x_2y_2)-1.(x_1y_1+x_1y_2-x_1y_1-x_2y_1)+1.(x_1y_2-x_2y_1)=0$

$\Rightarrow x_1y_2=x_2y_1$.

07. (a) When the square matrix $A=\left[a_{ij}\right]$ is skew-symmetric matrix, then $A^T=-A$ i.e., $a_{ij}=-a_{ji}$.

Clearly for the given matrix A, we will have $a_{13}=-a_{31}$; $a_{23}=-a_{32}$ i.e., $c=-2$; $-b=-3$.

That is, $b=3$; $c=-2$.

Also all the diagonal elements in a skew-symmetric matrix are always 0, i.e., $a_{22}=a=0$.

So, $a+b+c=0+3-2=1$.

08. (c) $P(A)=1-P(\bar{A})=\frac{1}{2}$; $P(B)=1-P(\bar{B})=\frac{1}{3}$

Also, $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}=\frac{7}{12}$

Now $P\left(\frac{\bar{A}}{\bar{B}}\right)=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}=\frac{P(\overline{A\cup B})}{P(\bar{B})}=\frac{1-P(A\cup B)}{P(\bar{B})}=\frac{1-\frac{7}{12}}{\frac{2}{3}}=\frac{5}{8}$.

09. (b) For obtuse angle, $\cos\theta<0$ i.e., $\frac{\vec{p}.\vec{q}}{|\vec{p}||\vec{q}|}<0 \Rightarrow \vec{p}.\vec{q}<0$

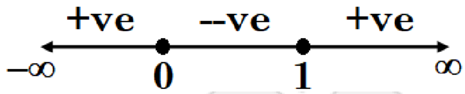
So, $(2\alpha^2\hat{i}-3\alpha\hat{j}+\hat{k}).(\hat{i}+\hat{j}+\alpha\hat{k})<0$

$\Rightarrow 2\alpha^2-3\alpha+\alpha<0$

$\Rightarrow 2\alpha^2-2\alpha<0$

$\Rightarrow 2\alpha(\alpha-1)<0$

$\Rightarrow \alpha\in(0, 1)$.



10. (c) $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2\left[|\vec{a}|^2+|\vec{b}|^2\right]=2(9+16)=50$

$\Rightarrow 5^2+|\vec{a}-\vec{b}|^2=50 \quad \Rightarrow |\vec{a}-\vec{b}|^2=50-25=25$

Hence, $|\vec{a}-\vec{b}|=5$

Alternatively, if we consider a rectangle whose adjacent sides are denoted by $\vec{a}$ and $\vec{b}$. Clearly, the diagonals will be given by vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. Also the diagonals in a rectangle are of same length. Hence, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|=5$. Note that, $|\vec{a}|^2+|\vec{b}|^2=3^2+4^2=5^2=|\vec{a}+\vec{b}|^2$.

11. (b)

Corner Points	Value of the objective function $Z=4x+3y$
O(0, 0)	0
R(40, 0)	160
Q(30, 20)	180 ← Maximum value
P(0, 40)	120

Since the feasible region is bounded so the maximum value of the objective function $Z=180$ is obtained at $Q(30, 20)$.

12. (a) Let $I=\int\frac{dx}{x^3(1+x^4)^{\frac{1}{2}}}=\frac{dx}{x^3\left[x^4(x^{-4}+1)\right]^{\frac{1}{2}}}=\int\frac{dx}{x^5\left(\frac{1}{x^4}+1\right)^{\frac{1}{2}}}$

Put $1+\frac{1}{x^4}=t \Rightarrow -4x^{-5}dx=dt \Rightarrow \frac{dx}{x^5}=-\frac{dt}{4}$

$\therefore I=-\frac{1}{4}\int\frac{dt}{t^{\frac{1}{2}}}=-\frac{1}{4}\times2\sqrt{t}+c$, where 'c' denotes any arbitrary constant of integration.

Hence, $I=-\frac{1}{2}\sqrt{1+\frac{1}{x^4}}+c=-\frac{1}{2x^2}\sqrt{1+x^4}+c$.

13. (a) We know, $\int_0^{2a} f(x)\,dx = 0$, if $f(2a-x) = -f(x)$

Let $f(x) = \text{cosec}^7 x$.

Now $f(2\pi - x) = \text{cosec}^7(2\pi - x) = -\text{cosec}^7 x = -f(x)$.

$\therefore \int_0^{2\pi} \text{cosec}^7 x\,dx = 0$.

14. (b) The given differential equation is $e^{\frac{dy}{dx}} = x$

Taking logarithm on both the sides, we get $\log e^{\frac{dy}{dx}} = \log x$

$\Rightarrow \frac{dy}{dx} \times \log e = \log x$ [$\because \log e = 1$

So, $dy = \log x\,dx$

$\Rightarrow \int dy = \int \log x\,dx$

$\Rightarrow y = \log x \int 1\,dx - \int\left(\frac{d}{dx}[\log x]\int 1\,dx\right)dx$ [Using by parts method

$\Rightarrow y = x\log x - \int \frac{1}{x} \times x\,dx$

So, $y = x\log x - x + c$.

15. (b) The graph represents $y = \cos^{-1} x$, whose domain is $x \in [-1, 1]$ and range is $y \in [0, \pi]$.

16. (d) Since the inequality $18x + 10y < 134$ has **no point in common with the feasible region** hence, the minimum value of the objective function $Z = 18x + 10y$ is 134 at $P(3, 8)$.

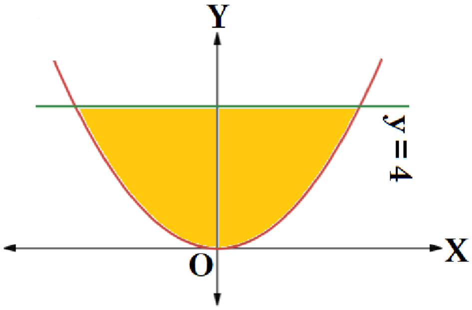
17. (d) The graph of the function $f : \mathbb{R} \to Z$ defined by $f(x) = [x]$ is a straight line for all $x \in (2.5 - h, 2.5 + h)$ where 'h' is an infinitesimally small positive quantity. Hence, the function is continuous and differentiable at $x = 2.5$.

18. (b) Consider the diagram given below.

The required region by the curve $x^2 = 4y$ is symmetrical about by y-axis.

So, required area (in sq. units) $= 2\int_0^4 2\sqrt{y}\,dy$

$$= 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{4} = \frac{64}{3} \text{ Sq. units.}$$



19. (a) Both (**A**) and (**R**) are true and (**R**) is the correct explanation of (**A**).

20. (a) Both (**A**) and (**R**) are true and (**R**) is the correct explanation of (**A**).

SECTION B

21. $\cot^{-1}(3x+5) > \frac{\pi}{4}$

$\Rightarrow \cot^{-1}(3x+5) > \cot^{-1} 1$

$\Rightarrow 3x + 5 < 1$ [$\because \cot^{-1} x$ is strictly decreasing function in its domain

$\Rightarrow 3x < -4$

$\Rightarrow x < -\dfrac{4}{3}$

$\therefore x \in \left(-\infty, -\dfrac{4}{3}\right).$

22. Given that $C(x) = 0.00013x^3 + 0.002x^2 + 5x + 2200$

The marginal cost function is $C'(x) = 0.00039x^2 + 0.004x + 5.$

Hence, $C'(150) = 0.00039 \times (150)^2 + 0.004 \times (150) + 5 = 14.375$ (in rupees).

23. Let $y = \tan^{-1} x$ and $z = \log x$.

Then $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$ and, $\dfrac{dz}{dx} = \dfrac{1}{x}$.

Now $\dfrac{dy}{dz} = \dfrac{\frac{dy}{dx}}{\frac{dz}{dx}}$

So, $\dfrac{dy}{dz} = \dfrac{\frac{1}{1+x^2}}{\frac{1}{x}} = \dfrac{x}{1+x^2}$.

OR

Let $y = (\cos x)^x$. Then, $y = e^{\log_e (\cos x)^x} = e^{x \log_e (\cos x)}$.

On differentiating both sides with respect to x, we get : $\dfrac{dy}{dx} = e^{x\log_e(\cos x)} \dfrac{d}{dx}\left[x \log_e(\cos x)\right]$

$\Rightarrow \dfrac{dy}{dx} = (\cos x)^x \left\{\log_e(\cos x) \times \dfrac{d}{dx}(x) + x \times \dfrac{d}{dx}\left[\log_e(\cos x)\right]\right\}$

$\Rightarrow \dfrac{dy}{dx} = (\cos x)^x \left\{\log_e(\cos x) \times 1 + x \times \dfrac{1}{\cos x} \times (-\sin x)\right\}$

$\Rightarrow \dfrac{dy}{dx} = (\cos x)^x \left[\log_e(\cos x) - x\tan x\right].$

24. We have $\vec{b} + \lambda\vec{c} = -\hat{i} + 2\hat{j} + \hat{k} + \lambda(3\hat{i} + \hat{j}) = (-1+3\lambda)\hat{i} + (2+\lambda)\hat{j} + \hat{k}$

Since $(\vec{b} + \lambda\vec{c}) \perp \vec{a}$ so, $(\vec{b} + \lambda\vec{c}).\vec{a} = 0$

$\Rightarrow \{(-1+3\lambda)\hat{i} + (2+\lambda)\hat{j} + \hat{k}\}.(2\hat{i} + 2\hat{j} + 3\hat{k}) = 0$

$\Rightarrow 2(-1+3\lambda) + 2(2+\lambda) + 3 = 0 \qquad \therefore \lambda = -\dfrac{5}{8}.$

OR

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = (4\hat{i} + 3\hat{k}) - \hat{k} = 4\hat{i} + 2\hat{k} = \vec{r}$ *say.*

Now $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{4}{2\sqrt{5}}\hat{i} + \dfrac{2}{\sqrt{5}}\hat{k} = \dfrac{2}{\sqrt{5}}\hat{i} + 0\hat{j} + \dfrac{1}{\sqrt{5}}\hat{k}$.

So, the angles made by the vector $\vec{r}$ with the x, y and the z-axes are respectively $\cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right)$, $\cos^{-1} 0 = \dfrac{\pi}{2}$ and, $\cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$.

Note that, a unit vector can be expressed as $\cos\alpha\,\hat{i}+\cos\beta\,\hat{j}+\cos\gamma\,\hat{k}$.

25. Let $\vec{a}=2\hat{i}-4\hat{j}-5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}+3\hat{k}$.

Then $\vec{d}_1=\vec{a}+\vec{b}=4\hat{i}-2\hat{j}-2\hat{k}$, $\vec{d}_2=\vec{a}-\vec{b}=-6\hat{j}-8\hat{k}$ are the diagonals of the parallelogram.

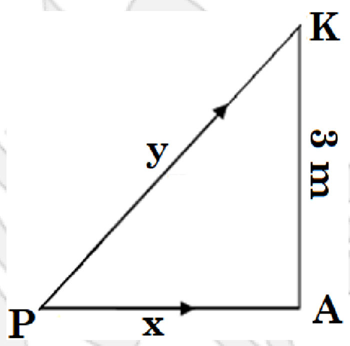
Now, area of the parallelogram $=\frac{1}{2}|\vec{d}_1\times\vec{d}_2|=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -2 & -2\\ 0 & -6 & -8\end{vmatrix}=\frac{1}{2}|4\hat{i}+32\hat{j}-24\hat{k}|=2|\hat{i}+8\hat{j}-6\hat{k}|$

$=2\sqrt{101}$ Sq. units.

Note that, if the second diagonal is taken as $\vec{d}_2=\vec{b}-\vec{a}$, then too it shall be correct.

SECTION C

26. Let at an instant, the kite is flying at K and the person who is flying the kite is at the point P.
Clearly, PK means the length of string at this instant; whereas AK means the height of kite.
Suppose PA be x metres.



So, $\frac{dx}{dt}=200$ cm/s (given).

In ΔKAP, $x^2+3^2=y^2$ ...(i)

$\Rightarrow 2x\frac{dx}{dt}+0=2y\frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt}=\frac{x}{y}\times\frac{dx}{dt}$

$\therefore \left.\frac{dy}{dt}\right]_{\text{at } y=5\text{ m}}=\frac{4}{5}\times 200=160$ cm/s.

[By (i), when $y=5$ m, $x=4$ m

So, the rate at which the string is being released is 160 cm/s.

27. Given that $A=\frac{1}{3}\sqrt{t}$

$\therefore \frac{dA}{dt}=\frac{1}{6\sqrt{t}}$ and, $\frac{d^2A}{dt^2}=-\frac{1}{12t\sqrt{t}}$.

Clearly $\frac{d^2A}{dt^2}<0\ \forall\ t\in(5,18)$.

This means that the rate of change of the ability to understand spatial concepts decreases (slows down) with the age.

28. Since $\theta=\cos^{-1}\left[\frac{\vec{l}_1.\vec{l}_2}{|\vec{l}_1||\vec{l}_2|}\right]$ so, $\theta=\cos^{-1}\frac{(\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-2\hat{j}+\hat{k})}{|(\hat{i}-2\hat{j}+3\hat{k})||(3\hat{i}-2\hat{j}+\hat{k})|}$

$\Rightarrow \theta=\cos^{-1}\left(\frac{3+4+3}{\sqrt{1+4+9}\sqrt{9+4+1}}\right)=\cos^{-1}\left(\frac{10}{14}\right)$

$\therefore \theta=\cos^{-1}\left(\frac{5}{7}\right)$.

Also, the Scalar projection of $\vec{l}_1$ on $\vec{l}_2=\vec{l}_1.\frac{\vec{l}_2}{|\vec{l}_2|}=(\hat{i}-2\hat{j}+3\hat{k}).\frac{(3\hat{i}-2\hat{j}+\hat{k})}{|(3\hat{i}-2\hat{j}+\hat{k})|}=\frac{3+4+3}{\sqrt{9+4+1}}=\frac{10}{\sqrt{14}}$.

OR

Note that a line which is perpendicular to the lines given as $\vec{r}=2\hat{i}+\hat{j}-3\hat{k}+\lambda(\hat{i}+2\hat{j}+5\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}-7\hat{k}+\mu(3\hat{i}-2\hat{j}+5\hat{k})$ has a vector parallel to it, which can be obtained by $\vec{b}=\vec{b}_1\times\vec{b}_2$.

That is, $\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 5\\ 3 & -2 & 5\end{vmatrix}=20\hat{i}+10\hat{j}-8\hat{k}$.

$\therefore$ Equation of required line passing through the point (–1, 2, 7) in the vector form is given by $\vec{r}=-\hat{i}+2\hat{j}+7\hat{k}+u(10\hat{i}+5\hat{j}-4\hat{k})$.

Also, the equation of line in Cartesian form is $\frac{x+1}{10}=\frac{y-2}{5}=\frac{z-7}{-4}$.

29. $\int\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx=\int\frac{1}{\log x}\times 1\,dx-\int\frac{1}{(\log x)^2}dx$

$$=\frac{1}{\log x}\int 1\,dx-\int\left\{\frac{d}{dx}\left(\frac{1}{\log x}\right)\int 1\,dx\right\}dx-\int\frac{1}{(\log x)^2}dx$$

$$=\frac{x}{\log x}+\int\frac{1}{(\log x)^2\times x}\times x\,dx-\int\frac{1}{(\log x)^2}dx$$

$$=\frac{x}{\log x}+\int\frac{1}{(\log x)^2}dx-\int\frac{dx}{(\log x)^2}$$

$=\frac{x}{\log x}+c$; where ‘c’ is any arbitrary constant of integration.

OR

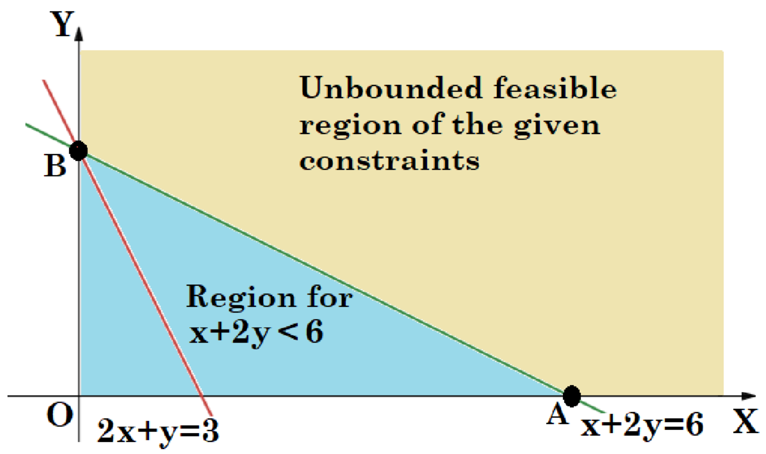
$\int_0^1 x(1-x)^n dx=\int_0^1(1-x)\{1-(1-x)\}^n dx$ $\left[\text{Using }\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right.$

$$=\int_0^1(1-x)x^n dx=\int_0^1 x^n dx-\int_0^1 x^{n+1}dx$$

$$=\frac{1}{n+1}\left[x^{n+1}\right]_0^1-\frac{1}{n+2}\left[x^{n+2}\right]_0^1$$

$$=\frac{1}{n+1}-\frac{1}{n+2}=\frac{1}{(n+1)(n+2)}.$$

30. The feasible region determined by the constraints $2x+y\ge 3$, $x+2y\ge 6$, $x\ge 0$, $y\ge 0$ has been shown below.



The corner points of unbounded feasible region are A(6, 0) and B(0, 3).

The values of $Z = x + 2y$ at these corner points are as follows.

Corner point	Value of the objective function
A(6, 0)	6
B(0, 3)	6

Since the feasible region is unbounded. It means, $Z = 6$ may or may not be the minimum value of Z. To check, let $x + 2y < 6$.

We observe that the region $x + 2y < 6$ has **no point in common with the unbounded feasible region**. Hence the minimum value of $Z = 6$.

It can be seen that the value of Z at points A and B is same.

When we take any other point on the line-segment joining the points A and B i.e., on the line $x + 2y = 6$ if we take a point $(2, 2)$ *say*, then also $Z = 6$. Note that, there will be **infinite such points** on the line $x + 2y = 6$ for which $Z = 6$.

Thus, the minimum value of Z occurs for more than two points, and it is equal to 6.

31. Since the event of raining today and not raining today are complementary events.

So if the probability that it rains today is 0.4, then the probability that it does not rain today is $1 - 0.4 = 0.6$ i.e., $P_1 = 0.6$.

If it rains today, the probability that it will rain tomorrow is 0.8, then the probability that it will not rain tomorrow is $1 - 0.8 = 0.2$ i.e., $P_2 = 0.2$.

If it does not rain today, the probability that it will rain tomorrow is 0.7 i.e., $P_3 = 0.7$; then the probability that it will not rain tomorrow is $1 - 0.7 = 0.3$ i.e., $P_4 = 0.3$.

(i) $P_1 \times P_4 - P_2 \times P_3 = 0.6 \times 0.3 - 0.2 \times 0.7 = 0.04$.

(ii) Let E_1 and E_2 be the events that it will rain today and it will not rain today respectively.

Then $P(E_1) = 0.4$ and $P(E_2) = 0.6$.

Let A be the event that it will rain tomorrow.

Then $P(A | E_1) = 0.8$ and $P(A | E_2) = 0.7$.

We have, $P(A) = P(E_1)P(A | E_1) + P(E_2)P(A | E_2)$

$\Rightarrow P(A) = 0.4 \times 0.8 + 0.6 \times 0.7 = 0.74$.

The probability of raining tomorrow is 0.74.

OR

Given $P(X = r) \propto \frac{1}{5^r}$.

Let $P(X = r) = k \times \frac{1}{5^r}$, where k is a positive constant.

Then $P(r = 0) = k \times \frac{1}{5^0}$, $P(r = 1) = k \times \frac{1}{5^1}$, $P(r = 2) = k \times \frac{1}{5^2}$, $P(r = 3) = k \times \frac{1}{5^3}$, ...

As $P(X = 0) + P(X = 1) + P(X = 2) + ... = 1$

$$\Rightarrow k\left(\frac{1}{5^0} + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3} + ...\right) = 1$$

$$\Rightarrow k\left(1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + ...\right) = 1$$

Note that, $1 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + ...$ is an infinite geometric progression, where $a = 1$, $r = \frac{1}{5}$.

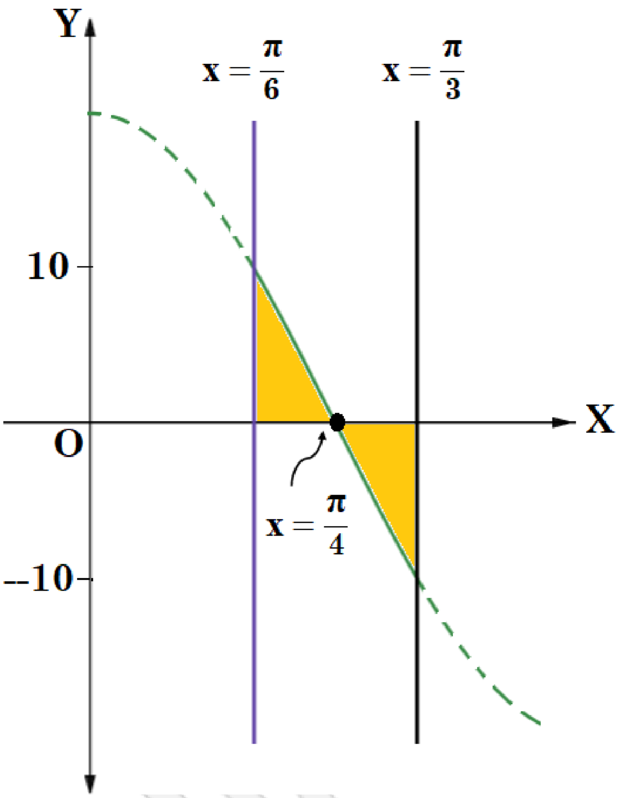
Using $S_\infty = \frac{a}{1-r}$, we get $k\left(\frac{1}{1-\frac{1}{5}}\right)=1$ $\quad \therefore k = \frac{4}{5}$.

Hence, $P(X<3) = P(X=0)+P(X=1)+P(X=2)$

$$= \frac{4}{5}\left(1+\frac{1}{5}+\frac{1}{5^2}\right) = \frac{4}{5}\left(\frac{25+5+1}{25}\right) = \frac{124}{125}.$$

SECTION D

32. Consider the following rough sketch of the curve $y = 20\cos 2x$; where $\frac{\pi}{6} \le x \le \frac{\pi}{3}$.



Now, the Required area $= \int_{\pi/6}^{\pi/4} 20\cos 2x\, dx + \left|\int_{\pi/4}^{\pi/3} 20\cos 2x\, dx\right|$

$$= 20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} + \left|20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}\right|$$

$$= 10\left(1-\frac{\sqrt{3}}{2}\right) + \left|10\left(\frac{\sqrt{3}}{2}-1\right)\right|$$

$$= 10\left(1-\frac{\sqrt{3}}{2}\right) + 10\left(1-\frac{\sqrt{3}}{2}\right)$$

$$= 20\left(1-\frac{\sqrt{3}}{2}\right) \text{ Sq. units}.$$

33. Note that, the points (2, 15), (4, 25) and (14, 15) shall satisfy $y = ax^2 + bx + c$. So, we get $15 = 4a + 2b + c$, $25 = 16a + 4b + c$ and $15 = 196a + 14b + c$.

The set of above linear equations can be represented in the matrix form as $AX = B$,

where $A = \begin{bmatrix} 4 & 2 & 1 \\ 16 & 4 & 1 \\ 196 & 14 & 1 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ and $B = \begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix}$.

Now $|A| = 4(4-14) - 2(16-196) + (224-784) = -40 + 360 - 560 = -240 \neq 0 \quad \therefore A^{-1}$ exists.

Now, $\text{adj.}(A) = \begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix} \qquad \therefore A^{-1} = \dfrac{\text{adj.}(A)}{|A|} = \dfrac{1}{-240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}$

Since $AX = B$ implies, $X = A^{-1}B$

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{1}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} = -\frac{1}{48}\begin{bmatrix} 24 \\ -384 \\ -48 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ 8 \\ 1 \end{bmatrix}$$

$\therefore a = -\dfrac{1}{2},\ b = 8,\ c = 1$.

So, the equation of path traversed by the ball becomes $y = -\dfrac{1}{2}x^2 + 8x + 1$.

34. We have $f(x) = |x|^3 = \begin{cases} x^3, & \text{if } x \geq 0 \\ -x^3, & \text{if } x < 0 \end{cases}$.

Now LHD of $f(x)$ at $x = 0$: $\displaystyle\lim_{x\to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0^-}\left(\frac{-x^3 - 0}{x}\right) = \lim_{x\to 0^-}(-x^2) = 0$

RHD of $f(x)$ at $x = 0$: $\displaystyle\lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0^+}\left(\frac{x^3 - 0}{x}\right) = \lim_{x\to 0^+}(x^2) = 0$.

$\because Lf'(0) = Rf'(0)$ so, $f(x)$ is differentiable at $x = 0$ and hence the derivative of $f(x)$ is given by $f'(x) = \begin{cases} 3x^2, & \text{if } x \geq 0 \\ -3x^2, & \text{if } x < 0 \end{cases}$.

Now LHD of $f'(x)$ at $x = 0$: $\displaystyle\lim_{x\to 0^-} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x\to 0^-}\left(\frac{-3x^2 - 0}{x}\right) = \lim_{x\to 0^-}(-3x) = 0$

RHD of $f'(x)$ at $x = 0$: $\displaystyle\lim_{x + 0^+} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x + 0^+}\left(\frac{3x^2 - 0}{x - 0}\right) = \lim_{x + 0^+}(3x) = 0$.

$\because$ LHD of $f'(x)$ at $x = 0$ and RHD of $f'(x)$ at $x = 0$ are both same.

So, $f'(x)$ is differentiable at $x = 0$.

Hence, $f''(x) = \begin{cases} 6x, & \text{if } x \geq 0 \\ -6x, & \text{if } x < 0 \end{cases}$.

OR

We have $(x-a)^2+(y-b)^2=c^2 \ldots(i)$

On differentiating w.r.t. x, we get $2(x-a)(1-0)+2(y-b)(y'-0)=0$

$\Rightarrow (x-a)=-(y-b)y' \ldots(ii)$

Again differentiating w.r.t. x, we get $-1=(y-b)y''+(y')^2$

$\Rightarrow -\dfrac{[1+(y')^2]}{y''}=(y-b)$

Replacing value of $(y-b)$ in (ii), $(x-a)=\dfrac{[1+(y')^2]}{y''}\times y'$

Substituting values of $(x-a)$ and $(y-b)$ in (i), we get :

$$\left(\frac{[1+(y')^2]}{y''}\times y'\right)^2+\left(-\frac{[1+(y')^2]}{y''}\right)^2=c^2$$

$$\Rightarrow \left([1+(y')^2]y'\right)^2+[1+(y')^2]^2=(cy'')^2$$

$$\Rightarrow \left[(y')^2+1\right][1+(y')^2]^2=(cy'')^2$$

$$\Rightarrow \frac{[1+(y')^2]^3}{y''^2}=c^2$$

$$\Rightarrow c=\frac{[1+(y')^2]^{\frac{3}{2}}}{y''}$$

$$\therefore c=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}.$$

Hence $c=\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of both 'a' and 'b'.

Alternatively, we have $(x-a)^2+(y-b)^2=c^2$, $c>0$.

Let $x-a=c\cos\theta$ and $y-b=c\sin\theta$.

Therefore, $\dfrac{dx}{d\theta}=-c\sin\theta$ and $\dfrac{dy}{d\theta}=c\cos\theta$.

$\therefore \dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}=-\cot\theta$.

On differentiating both sides with respect to x, we get $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=\dfrac{d}{dx}(-\cot\theta)$

$$\Rightarrow \frac{d^2y}{dx^2}=\text{cosec}^2\theta\times\frac{d\theta}{dx}=\text{cosec}^2\theta\times\frac{1}{-c\sin\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2}=-\frac{\text{cosec}^3\theta}{c}$$

Now $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}=\dfrac{\left[1+(-\cot\theta)^2\right]^{\frac{3}{2}}}{\dfrac{-\operatorname{cosec}^3\theta}{c}}$

$\Rightarrow \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}=\dfrac{c[1+\cot^2\theta]^{\frac{3}{2}}}{-\operatorname{cosec}^3\theta}=\dfrac{-c(\operatorname{cosec}^2\theta)^{\frac{3}{2}}}{\operatorname{cosec}^3\theta}=-c$, which is constant and is independent of both a and b.

35. Given equation of lines are $\vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\lambda(7\hat{i}-6\hat{j}+\hat{k})$ and $\vec{r}=(3\hat{i}+5\hat{j}+7\hat{k})+\mu(\hat{i}-2\hat{j}+\hat{k})$.

So $\vec{a}_1=-\hat{i}-\hat{j}-\hat{k},\ \vec{b}_1=7\hat{i}-6\hat{j}+\hat{k};\ \vec{a}_2=3\hat{i}+5\hat{j}+7\hat{k},\ \vec{b}_2=\hat{i}-2\hat{j}+\hat{k}$.

Now $\vec{a}_2-\vec{a}_1=4\hat{i}+6\hat{j}+8\hat{k},\ \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}=-4\hat{i}-6\hat{j}-8\hat{k}$.

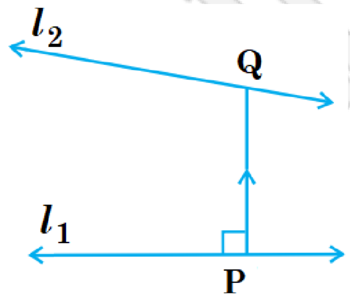
Hence, $\text{S.D.}=\dfrac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}=\dfrac{\left|(4\hat{i}+6\hat{j}+8\hat{k}).(-4\hat{i}-6\hat{j}-8\hat{k})\right|}{\sqrt{16+36+64}}$

$\Rightarrow \text{S.D.}=\dfrac{4\times\left|(2\hat{i}+3\hat{j}+4\hat{k}).(-2\hat{i}-3\hat{j}-4\hat{k})\right|}{\sqrt{116}}$

$\Rightarrow \text{S.D.}=\dfrac{4\times|-4-9-16|}{2\sqrt{29}}=\dfrac{58}{\sqrt{29}}=2\sqrt{29}$ units.

Alternatively, Given that equation of both the lines are $\vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\lambda(7\hat{i}-6\hat{j}+\hat{k})$ ...(i) and $\vec{r}=(3\hat{i}+5\hat{j}+7\hat{k})+\mu(\hat{i}-2\hat{j}+\hat{k})$ ...(ii)

The given lines are non-parallel lines as the vectors $7\hat{i}-6\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ are not parallel.



There is a unique line segment PQ, P lying on line (i) and Q on the other line (ii), which is at right angles to both the lines. PQ is the shortest distance between the lines.

Hence, the shortest possible distance between the lines $=$ PQ.

Let the position vector of the point P lying on the line $\vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\lambda(7\hat{i}-6\hat{j}+\hat{k})$ where 'λ' is scalar, is $(7\lambda-1)\hat{i}-(6\lambda+1)\hat{j}+(\lambda-1)\hat{k}$, for some λ and the position vector of the point Q lying on the other line given by $\vec{r}=(3\hat{i}+5\hat{j}+7\hat{k})+\mu(\hat{i}-2\hat{j}+\hat{k})$ where 'μ' is a scalar, is $(\mu+3)\hat{i}+(-2\mu+5)\hat{j}+(\mu+7)\hat{k}$, for some μ.

Now the vector $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=(\mu+3-7\lambda+1)\hat{i}+(-2\mu+5+6\lambda+1)\hat{j}+(\mu+7-\lambda+1)\hat{k}$

i.e., $\overrightarrow{PQ}=(\mu-7\lambda+4)\hat{i}+(-2\mu+6\lambda+6)\hat{j}+(\mu-\lambda+8)\hat{k}$.

Since $\overrightarrow{PQ}$ is perpendicular to both the lines (i) and (ii), so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $7\hat{i}-6\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$.

Therefore, $(\mu-7\lambda+4).7+(-2\mu+6\lambda+6).(-6)+(\mu-\lambda+8).1=0$

and, $(\mu-7\lambda+4).1+(-2\mu+6\lambda+6).(-2)+(\mu-\lambda+8).1=0$ $\quad$ [$\because \vec{a}\perp\vec{b}$ implies $\vec{a}.\vec{b}=0$

$\Rightarrow 10\mu-43\lambda=0$ and, $3\mu-10\lambda=0$

On solving the above equations, we get $\mu=\lambda=0$.

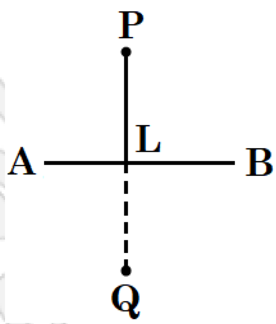
So, the position vector of the points P and Q are $-\hat{i}-\hat{j}-\hat{k}$ and $3\hat{i}+5\hat{j}+7\hat{k}$ respectively.

Hence, $\overrightarrow{PQ}=4\hat{i}+6\hat{j}+8\hat{k}$.

Therefore, required shortest distance is $\left|\overrightarrow{PQ}\right|=\sqrt{4^2+6^2+8^2}=\sqrt{116}=2\sqrt{29}$ units.

OR

Let $P(1, 2, 1)$ be the given point and L be the foot of the perpendicular from P to the given line AB (as shown in the figure).



Let's put $\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-1}{3}=\lambda$.

Then $x=\lambda+3,\ y=2\lambda-1,\ z=3\lambda+1$.

Let the coordinates of the point L be $(\lambda+3,\ 2\lambda-1,\ 3\lambda+1)$.

So, the direction ratios of PL are given by

$(\lambda+3-1,\ 2\lambda-1-2,\ 3\lambda+1-1)$ i.e., $(\lambda+2,\ 2\lambda-3,\ 3\lambda)$.

The direction ratios of the given line AB are $1,\ 2$ and 3.

The line AB is perpendicular to PL.

Therefore, we have $(\lambda+2).1+(2\lambda-3).2+(3\lambda).3=0$

$\Rightarrow \lambda+2+4\lambda-6+9\lambda=0$

$\Rightarrow 14\lambda-4=0$

$\Rightarrow \lambda=\frac{2}{7}$.

Therefore, coordinates of the point L are $\left(\frac{23}{7},\ -\frac{3}{7},\ \frac{13}{7}\right)$.

Let $Q(x_1,\ y_1,\ z_1)$ be the image of $P(1, 2, 1)$ with respect to the given line.

Then L is the mid-point of PQ.

Therefore, $\frac{x_1+1}{2}=\frac{23}{7},\ \frac{y_1+2}{2}=-\frac{3}{7},\ \frac{z_1+1}{2}=\frac{13}{7}$

$\Rightarrow x_1=\frac{39}{7},\ y_1=-\frac{20}{7},\ z_1=\frac{19}{7}$

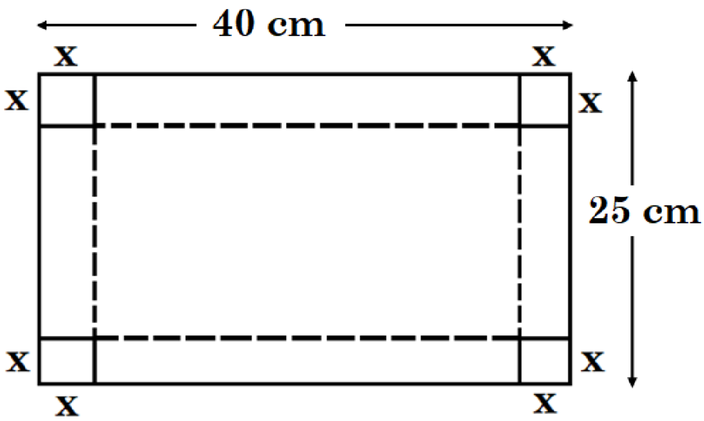
Hence, the image of the point $P(1, 2, 1)$ with respect to the given line is $Q\left(\frac{39}{7},\ -\frac{20}{7},\ \frac{19}{7}\right)$.

Also the equation of the line joining $P(1, 2, 1)$ and $Q\left(\frac{39}{7},\ -\frac{20}{7},\ \frac{19}{7}\right)$ is given by

$$\frac{x-1}{\frac{39}{7}-1}=\frac{y-2}{-\frac{20}{7}-2}=\frac{z-1}{\frac{19}{7}-1} \text{ i.e., } \frac{x-1}{\frac{32}{7}}=\frac{y-2}{-\frac{34}{7}}=\frac{z-1}{\frac{12}{7}} \text{ i.e., } \frac{x-1}{16}=\frac{y-2}{-17}=\frac{z-1}{6}.$$

SECTION E

36. **(i)** Consider the diagram shown.



For the container that is made by folding up the flaps, the length of container is $(40-2x)$ cm, width is $(25-2x)$ cm and height is x cm.

Hence, the volume of container is given by $V=\left[(40-2x)(25-2x)x\right]\text{ cm}^3$

$\therefore\ V=2(2x^3-65x^2+500x)\text{ cm}^3$.

(ii) $\frac{dV}{dx}=2(6x^2-130x+500)=4(3x^2-65x+250)=4(3x-50)(x-5)$.

(iii) For extreme values, $\frac{dV}{dx}=4(3x-50)(x-5)=0 \Rightarrow x=5$ $\left[\because x\neq\frac{50}{3}\right.$

We have $\frac{d^2V}{dx^2}=24x-260$

$\because \left(\frac{d^2V}{dx^2}\right)_{\text{at } x=5}=-140<0$.

$\therefore$ V is maximum when $x=5$ cm.

OR

(iii) Since $\frac{dV}{dx}=4(3x^2-65x+250)$, so we get $\frac{d^2V}{dx^2}=4(6x-65)$.

Note that $\frac{dV}{dx}$ at $x=\frac{65}{6}$ exists and, $\frac{d^2V}{dx^2}$ at $x=\frac{65}{6}$ is 0.

Also $\frac{d^2V}{dx^2}$ at $x=\left(\frac{65}{6}\right)^-$ is negative and $\frac{d^2V}{dx^2}$ at $x=\left(\frac{65}{6}\right)^+$ is positive.

$\therefore x=\frac{65}{6}$ is a point of inflection.

Note that, the second derivative of the function changes its sign from negative to positive or vice-versa at the point of inflection.

37. **(i)** Number of relations from B to G is equal to the number of subsets of $B\times G$

$$=2^{n(B\times G)}$$
$$=2^{n(B)\times n(G)}$$
$$=2^{3\times 2}=2^6=64.$$

(ii) Smallest Equivalence relation on G is $\{(g_1, g_1), (g_2, g_2)\}$.

(iii) (A) For reflexive but not symmetric R_1, we must have

$R_1=\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)\}$.

So the minimum ordered pairs to be added are $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$.

Note that, it can be any one of the pair from, $(b_3, b_2), (b_1, b_3), (b_3, b_1)$ in place of the pair (b_2, b_3) also.

(B) For reflexive and symmetric but not transitive R_1, we must have
$R_1=\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)\}$.
So the minimum ordered pairs to be added in the relation R_1 are $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$.
Note that, $(b_3, b_2)\in R_1$ and $(b_2, b_1)\in R_1$ but $(b_3, b_1)\notin R_1$. So, R_1 is not transitive.

OR

(iii) For the given curve $x^2=4y$, let $y=f(x)=\frac{x^2}{4}$.

Let $x_1, x_2\in[0, 20\sqrt{2}]$ such that $f(x_1)=f(x_2)$

Then $\frac{x_1^2}{4}=\frac{x_1^2}{4}$

$\Rightarrow x_1^2=x_2^2$

$\Rightarrow (x_1-x_2)(x_1+x_2)=0$

$\Rightarrow x_1=x_2$ [Since $x_1, x_2\in[0, 20\sqrt{2}]$ so, $x_1+x_2\neq 0$.

$\therefore f$ is one-one function.

As $y=\frac{x^2}{4}$ $\Rightarrow x=2\sqrt{y}$

Now $0\le y\le 200$ hence the value of y is non-negative and $f(x)=f(2\sqrt{y})=\frac{(2\sqrt{y})^2}{4}=y$.

$\therefore$ For any arbitrary $y\in[0, 200]$, the pre-image of y exists in $[0, 20\sqrt{2}]$.
Hence f is onto function.

38. Let E_1 be the event that the parrot and the owl fly from Cage-I,
E_2 be the event that two parrots fly from Cage-I,
E_3 be the event that the owl is still in Cage-I,
$\bar{E}_3$ be the event that the owl is not in Cage-I.

Now $n(E_1\cap E_3)=({}^5C_1\times{}^1C_1)({}^7C_1\times{}^1C_1)=5\times 7=35$,
$n(E_1\cap\bar{E}_3)=({}^5C_1\times{}^1C_1)({}^7C_2)=5\times 21=105$,
$n(E_2\cap E_3)=({}^5C_2)({}^8C_2)=10\times 28=280$ and, $n(E_2\cap\bar{E}_3)=0$.
So, total sample points will be $n(S)=35+105+280+0=420$.

(i) Probability that the owl is still in Cage-I $=P(E_3)=P(E_1\cap E_3)+P(E_2\cap E_3)$

$$=\frac{35+280}{420}=\frac{315}{420}=\frac{3}{4}.$$

(ii) The probability that one parrot and the owl flew from Cage-I to Cage-II given that the owl is still in Cage-I is $P\left(\frac{E_1}{E_3}\right)$.

Now using Bayes' Theorem, $P\left(\frac{E_1}{E_3}\right)=\frac{P(E_1\cap E_3)}{P(E_3)}=\frac{P(E_1\cap E_3)}{P(E_1\cap E_3)+P(E_2\cap E_3)}=\frac{\frac{35}{420}}{\frac{315}{420}}=\frac{1}{9}$.